**CALCULUS - PRACTICAL III - INTEGRATION ETC.**

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## 1. Definite Integral

Before proceeding, recall that in order to get help from Matlab, you can use either `help` or `doc` at the prompt.

To compute definite integrals, as it is a symbolic operation (unless one is performing an approximate computation), a symbolic variable is required, for which `syms` is used:

```
> syms x
```

and then one can perform the integration with respect to it, using `int`:

```
> int(cos(x).*sin(x),x,0,3)
```

will compute

$$\int_0^3 \cos(x)\sin(x)\,dx.$$

1.1. **Drill.** Compute the following definite integrals:

$$1)\ \int_{-1}^{1} \log(|x|)\,dx$$

$$2)\ \int_{-\pi}^{10} \cos(x)\,dx$$

$$3)\ \int_{1}^{2} e^x\,dx$$

$$4)\ \int_{e}^{\pi} \tan(x)\,dx$$

They can be done this way:

```
> syms t; int(log(abs(t)),t,-1,1)
> syms t; int(cos(t), t, -pi, 10)
> syms t; int(exp(t),t,1,2)
> syms u; int(tan(u),u, exp(1), pi)
```

Notice how the symbolic variable used is irrelevant.

## 2. Primitive (antiderivative)

In order to compute antiderivatives (primitive functions) one needs (for the same reason) a symbolic variable declared with `syms`. For example, to compute the antiderivative of $\cos(x)$:

```
> syms u
> int(cos(u),u)
```

(again, it is irrelevant to use either $x$ or $u$). One can also use anonymous functions:

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for example, one might compute the antiderivative of $P(t) = \sqrt{1-t^2}$, by means of the following commands:

```
> clear u
> P = @(x) sqrt(1-x.^2)
> syms u;% (we have cleared it)
> int(P(u),u)
```

In order to use the antiderivative as a numerical function, one should transform it using `matlabFunction`, as we did previously.

## 3. Improper integrals

Matlab can also compute improper integrals (when it can); if the integral is divergent, it shows a message, otherwise it gives the (approximate) value. In the divergent case, if the *Cauchy principal value* exists, it can also be computed.

For instance, the area between the $OX$ axis and the graph of $1/x^3$ from $x = 1$ to $x = \infty$ can be computed as follows:

```
> syms t; int(1/t.^3,t,1,inf)
```

and the area between the $OX$ axis and $1/\sqrt{x}$ from $x = 0$ to $x = 2$:

```
> syms u; int(1/u^(1/2), u, 0, 2)
```

The following improper integral is divergent but it has a Cauchy principal value (the command will work or not depending on the version of Matlab):

```
> syms u; int(1/u^3, u, -1, 1)
```

## 4. Exercises

(As always, use a single file for each exercise and separate parts by comments).

**Exercise 1.** Compute the following definite integrals:

$$\int_{-2}^{3} \sin(u-2)\,du$$
$$\int_{-10}^{-9} \frac{1}{1+(t-1)^4}\,dt$$
$$\int_{6}^{8} \frac{x^2+1}{1-2x+2}\,dx$$
$$\int_{2}^{3} \sqrt{1-v^2}\,dv$$

**Exercise 2.** Compute the antiderivatives of the following functions:

$$h(u) = \log^2(u) + e^u$$
$$f(x) = \tan(x)$$
$$g(t) = \frac{t}{t^2-1}$$
$$s(x) = \frac{x^3}{x^2-2x+2}$$

and evalute them at $-1, 0, \pi$ and $e^2$.

**Exercise 3.** Plot the function $y(x) = x(1+\sin(x))e^{\frac{-x}{10}}$ for $x$ from 3 to 100 using 1000 points. Compute the area bounded by that graph and the $OX$ axis, from $x = 1$ to $x = \infty$, if it is finite.

**Exercise 4.** A spring in rest position is $1m$ long. A force of $100N$ compresses it to $0.9m$. How many Joules are required to compress it to half its normal length? What is the length of the spring after a work of $20J$ has been performed?