ALL ANALYTIC BRANCHES IN $(\mathbb{C}^n, 0)$ ARE TOPOLOGICALLY EQUIVALENT

P. FORTUNY AYUSO

Result. Two analytic branches in $(\mathbb{C}^n, 0)$ are topologically equivalent for $n > 2$.
Proof. Let γ be such an analytic branch. The intersection $\gamma_\epsilon = \gamma \cap \mathbb{S}_\epsilon$ of γ with the sphere of radius ϵ is obviously a real curve (it is actually smooth), a knot, which can be parametrized as $\gamma_\epsilon(\theta)$, for $\theta \in [0, 2\pi]$. This parametrization is continuous in ϵ and θ because it is the restriction to $\gamma^{-1}(\mathbb{S}_\epsilon)$ of the map γ. In spherical coordinates, this knot can be parametrized as

$$\gamma_\epsilon \equiv (\rho = \epsilon, \theta_1(\epsilon, \theta), \ldots, \theta_{2n-1}(\epsilon, \theta))$$

where $(\rho, \theta_1, \ldots, \theta_{2n-1})$ are the spherical coordinates on $\mathbb{S}_\epsilon$. The map above is continuous in ϵ, θ for obvious reasons. Now, for fixed ϵ, the knot

$$(\rho = \epsilon, \theta_1(\epsilon, \theta), \ldots, \theta_{2n-1}(\epsilon, \theta))$$

can be continuously unknotted (isotopically, the unknot being $\rho = \epsilon, \theta_1, 0, 0, \ldots, 0)$ with parameter θ_1) in a continuous way depending on ϵ (this should be more or less obvious), in the coordinates $\theta_1, \ldots, \theta_{2n-1}$, because $n > 2$. By construction, this isotopy leaves the origin fixed, and we are done.

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